Advanced Higher Applied Mathematics (Mechanics)

Unit 1

Outcome 1.1 Motion In A Straight Line	**NS**	**OT**	**VG**
I know the meaning of *position, displacement, velocity, acceleration, uniform speed, uniform acceleration, scalar quantity, vector quantity*.			
I can draw, interpret and use distance/time, velocity/time and acceleration/time graphs.			
I know the area under a velocity/time graph represents the distance travelled.			
I know the rates of change $v = \frac{dx}{dt} = \dot{x}$ and $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = \dot{v} = \ddot{x}$			
I can derive, by calculus, and use the equations for motion in a straight line with **constant acceleration**, namely $v = u + at, s = ut + \frac{1}{2}at^2$ and from these $v^2 = u^2 + 2as, s = \frac{(u+v)}{2}t$			
I can solve analytically problems involving motion in one dimension under constant acceleration, including vertical motion under constant gravity.			
I can solve problems involving motion in one dimension where the acceleration is dependent on time i.e. $a = \frac{dv}{dt} = f(t)$			

Outcome 1.2 Relative Position and Velocity	**NS**	**OT**	**VG**
I know the meaning of the terms *relative position, relative velocity and relative acceleration, air speed, ground speed* and *nearest approach*.			
I am familiar with notation for relative position, velocity and acceleration vectors of 2 objects.			
I can resolve vectors into components.			
I can differentiate and integrate vector functions in time.			
I can use position, velocity and acceleration vectors to solve practical problems.			
I can solve problems involving collision courses and nearest approach.			

Outcome 1.3 Motion of Projectiles in a Vertical Plane	**NS**	**OT**	**VG**
I know the meaning of the terms *projectile, velocity, angle of projection, trajectory, time of flight, range* and *constant gravity*.			
I can solve the vector equation $\ddot{r} = -gj$ to obtain **r** in terms of its horizontal and vertical components.			
I can obtain and solve the equations of motion $\ddot{x} = 0, \ddot{y} = -g$, obtaining expressions for $\dot{x}, \dot{y}, x$ and y in any particular case.			
I can find the time of flight, greatest height reached and the range of a projectile.			
I can find the maximum range of a projectile on a horizontal plane and the angle of projection to achieve this.			
I can find, and use, the equation of the trajectory of a projectile.			
I can solve problems in two-dimensional motion involving projectiles under a constant gravitational force and neglecting air resistance.			

Outcome 1.4 Forces and Newton's Laws of Motion	**NS**	**OT**	**VG**
I can understand the terms *mass, force, weight, momentum, balanced and unbalanced forces, resultant force, equilibrium* and *resistive forces*.			
I know Newton's first and third laws of motion.			
I can resolve forces in two dimensions to find their components.			
I can combine forces to find a resultant force.			
I can understand the concept of static and dynamic friction and limiting friction.			
I understand the terms frictional force, normal reaction, coefficient of friction μ , angle of friction λ , and know the equations $F = \mu R$ and $\mu = \tan\theta$.			
I can solve problems involving a particle or body in equilibrium under the action of certain forces.			
I know Newton's second law of motion, that force is the rate of change of momentum, and derive the equation $F = ma$.			
I can use this equation to form equations of motion to model practical problems on motion in a straight line.			
I can solve such equations modelling motion in one dimension, including cases where the acceleration is dependent on time.			
I can solve problems involving friction and problems on both rough and smooth inclined planes.			

Advanced Higher Applied Mathematics (Mechanics)

Unit 2

Outcome 2.1 Motion In A Horizontal Circle	**NS**	**OT**	**VG**
I know the meaning of *angular velocity and angular acceleration*.			
I know that for motion in a circle where θ = ωt then: $\mathbf{r} = r\cos(\omega t)\mathbf{i} + r\sin(\omega t)\mathbf{j}$ $\mathbf{v} = -r\omega \sin(\omega t)\mathbf{i} + r\omega \cos(\omega t)\mathbf{j}$ $\mathbf{a} = -r\omega^2 \cos(\omega t)\mathbf{i} - r\omega^2 \sin(\omega t)\mathbf{j}$.			
I know that, from the above,: $v = r\omega$, $a = \omega^2 r = v^2/r$ and $\mathbf{a} = -\omega\mathbf{r}$.			
I can apply the above equations to motion in a horizontal circle with uniform angular velocity including skidding, banking, conical pendulum and other applications.			
I know Newton's inverse square law of gravitation, namely $F \propto {}^1/r^2$			
I can apply Newton's inverse square law of gravitation to simplified examples of motion of satellites and moons for circular orbits only.			
I can find the time for one orbit and the height above the surface etc.			

Outcome 2.2 Relative Position and Velocity	**NS**	**OT**	**VG**
I know the meaning of the terms *relative position, relative velocity and relative acceleration, air speed, ground speed* and *nearest approach*.			
I am familiar with notation for relative position, velocity and acceleration vectors of 2 objects.			
I can resolve vectors into components.			
I can differentiate and integrate vector functions in time.			
I can use position, velocity and acceleration vectors to solve practical problems.			
I can solve problems involving collision courses and nearest approach.			

Outcome 1.3 Motion of Projectiles in a Vertical Plane	**NS**	**OT**	**VG**
I know the meaning of the terms *projectile, velocity, angle of projection, trajectory, time of flight, range* and *constant gravity*.			
I can solve the vector equation $\ddot{r} = -gj$ to obtain **r** in terms of			

its horizontal and vertical components.			
I can obtain and solve the equations of motion $\ddot{x} = 0, \ddot{y} = -g$, obtaining expressions for $\dot{x}, \dot{y}, x$ and y in any particular case.			
I can find the time of flight, greatest height reached and the range of a projectile.			
I can find the maximum range of a projectile on a horizontal plane and the angle of projection to achieve this.			
I can find, and use, the equation of the trajectory of a projectile.			
I can solve problems in two-dimensional motion involving projectiles under a constant gravitational force and neglecting air resistance.			

Outcome 1.4 Forces and Newton's Laws of Motion	**NS**	**OT**	**VG**
I can understand the terms *mass, force, weight, momentum, balanced and unbalanced forces, resultant force, equilibrium* and *resistive forces*.			
I know Newton's first and third laws of motion.			
I can resolve forces in two dimensions to find their components.			
I can combine forces to find a resultant force.			
I can understand the concept of static and dynamic friction and limiting friction.			
I understand the terms frictional force, normal reaction, coefficient of friction μ, angle of friction λ, and know the equations $F = \mu R$ and $\mu = \tan\theta$.			
I can solve problems involving a particle or body in equilibrium under the action of certain forces.			
I know Newton's second law of motion, that force is the rate of change of momentum, and derive the equation $F = ma$.			
I can use this equation to form equations of motion to model practical problems on motion in a straight line.			
I can solve such equations modelling motion in one dimension, including cases where the acceleration is dependent on time.			
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Unit 3

Outcome 1.1 Applying Algebraic Skills	**NS**	**OT**	**VG**
I know how to expand an expression of the form $(ax + by)^n$ using the binomial expansion where $n \leq 7$, using $(x+a)^n = \sum_{k=0}^{n} \binom{n}{k} x^k a^{n-k}$.			
I can express a proper rational function as a sum of partial fractions where the denominator is of the type: $\frac{7x+1}{x^2+x-6}$ (linear factors)			
I can express a proper rational function as a sum of partial fractions where the denominator is of the type: $\frac{5x^2-x+6}{x^3+3x}$ (irreducible quadratic factor)			
I can express a proper rational function as a sum of partial fractions where the denominator is of the type: $\frac{3x+10}{x^2+6x+9}$ (repeated factor)			
Reduce an improper rational function to a polynomial and a proper rational function by division or otherwise eg. $\frac{x^3+2x^2-2x+2}{(x-1)(x+3)}$			

Outcome 1.2 Applying Calculus Skills to Differentiation	**NS**	**OT**	**VG**
I can differentiate functions involving: tan *x*, sec *x*, cosec *x*, cot *x*.			
I can differentiate functions involving: e^x, ln *x*			
I can differentiate functions using the chain rule $\left(f\left(g(x)\right)\right)' = f'\left(g(x)\right).g'(x)$			
I can differentiate functions using the product rule $\left(f(x)g(x)\right)' = f'(x)g(x) + f(x)g'(x)$			
I can differentiate functions using the quotient rule $\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x)-f(x)g'(x)}{(g(x))^2}$			
I can differentiate functions which require more than one application of the chain rule, product rule or quotient rule			
I know that $\frac{dy}{dx} = \frac{1}{dx/dy}$			
I can apply differentiation to simple rates of change eg rectilinear motion, optimisation.			

I can use parametric differentiation to find the first and second derivatives.			
I can apply differentiation to related rates in problems where the functional relationship is given explicitly eg. motion in a plane.			
I can solve practical related rates by first establishing a functional relationship between appropriate variables.			
I can differentiate functions expressed implicitly eg. find $\frac{dy}{dt}$ given $\frac{dx}{dt}$ and the function $x^2 + y^2 = r^2$ and *x* and *y* are functions of *t*.			

Outcome 1.3 Applying Calculus Skills to Integration	**NS**	**OT**	**VG**
I know and can use standard results including $\int e^x dx$, $\int \frac{1}{x} dx$, $\int sec^2 x\ dx$			
I can integrate using a substitution when the substitution is given.			
I can integrate a simple product or quotient of functions when one function is the derivative of the other.			
I can integrate proper rational functions using partial fractions.			
I can use one or repeated applications of integration by parts.			
I can apply integration to a range of physical situations including to evaluate areas, volumes by revolution and the centre of mass of a uniform lamina bounded by curves.			

Outcome 1.4 Applying Calculus Skills to Differential Equations	**NS**	**OT**	**VG**
I can find a general solution of a first order differential equation where the variables can be separated.			
I can solve a linear first order differential equation using an Integrating Factor.			
I can solve second order homogeneous equations where the auxiliary equation has real roots.			
I can formulate a simple statement involving rate of change as a separable first order differential equation.			
I can find general solutions and solve initial value problems, for example, mixing problems, growth and decay problems, simple electronic circuits and simple examples of damped simple harmonic motion			